**Measure Theory and integration (Solution set 1)**

**1.We know that the statement exsixts has the following meaning.Negate the statement.**

(ƷA)(ᵿ t > 0) (Ʒ ɗ > 0 ) (ᵿ x) ( 0 < |x – a| < ɗ => |f(x) – A| < ϵ

Solution: (ᵿ A) ( Ʒ t >0) (ᵿɗ > 0) (Ʒ x) (0 < |x – a| <ɗ => |f(x) –A| < ϵ

**2.Write the following statement in symbol form and then form its negation , for each ϵ > 0 ,there exsists an N>0 s.t**

**| fn(x) – f(x) | < ϵ Whenever n>N and xϵS.**

Solution: ᵿ ϵ > 0 ,Ʒ N >0 s.t |fn(x) –f(x) | < ϵ

Negation: Ʒ ϵ >0 ,ᵿ N > 0 s.t |fn(x) – f(x) | > ϵ whenever n>N and x ϵS.

**3.Distinguish between two statements (ᵿ x ϵR)( Ʒ y ϵR) (x+ y = 0 ) and**

**( y ϵ R)( ᵿ x ϵ R) (x + Y = 0)**

Solution: Here (ᵿ x ϵR)( Ʒ y ϵR) (x+ y = 0 ) is true

( y ϵ R)( ᵿ x ϵ R) (x + Y = 0) is false

**4.Condider the following two statement**

**( Ʒ x ϵ R) (ᵿ yϵ s) (y < x), ( Ʒ x ϵ R) (ᵿ yϵ s) ( y > x) ,**

**(Ʒ rϵ R) (ᵿ yϵ s)(|y| < r) Determine which of these statements is true for each of the following choice of s. a . (s) = [-3,10) (b) s = Q**

Solution: ( Ʒ x ϵ R) (ᵿ yϵ s) (y < x)

a.s = [-3,10) is false

b. s = Q take any poin (1/2 , 1/3) ,hence is is true.

( Ʒ x ϵ R) (ᵿ yϵ s) ( y > x)

a.s = [-3 , 10) is true b. S = Q is false

(Ʒ rϵ R) (ᵿ yϵ s)(|y| < r)

a.s = [-3 , 10) is false b.s = Q is false

**5.Write converse inverse and contrapositive of the statement.**

**“ if x < 0 then x2 – x > 0 “,Also write the negation of each statement.**

Solution: Given statement is “ if x < 0 then x2 – x > 0 “.

Converse: if x2 – x > 0 then x < 0.

Nagation: x2 – x > 0 and x > 0

Inverse: if x > 0 then x2 – x < 0.

Negation: x > 0 and x2 – x > 0.

Contrapositive: if x2 – x < 0 then x > 0

Negation: x2 – x < 0 and x < 0.

**6. Let f : x -> y be a function .Then prove that**

**i. A1 С A2 => F(A1) С F(A2) ii. F(Ui Ai) = Ui F(Ai) iii.F(ᴒi Ai) С ᴒi f(Ai)**

Solution: i. . A1 С A2 taking F on both side F(A1) С F(A2)

There fore A1 С A2 => F(A1) С F(A2)

ii. F(Ui Ai) = Ui F(Ai) Let xϵ f(Ui Ai) ⬄ f-(x) ϵ UiAi ⬄ f-(x) ϵ Ai for some x ⬄ x ϵ f(A) for some x => x ϵ Ui F(Ai) for some x. Hence F(Ui Ai) = Ui F(Ai)

iii. .F(ᴒi Ai) С ᴒi f(Ai) Let x ϵ F(ᴒi Ai) => f-(x) ϵ (ᴒi Ai) for all x => f-(x) ϵ Ai for all x => x ϵ F(Ai) hence x ϵ ᴒi f(Ai) hence F(ᴒi Ai) С ᴒi f(Ai) .

**7. Let F : X -> Y be a function. If B is subset of y then its inverse image f-(B) is the subset of x define by**

**f-(B) = {x : f(x) ϵB} , Now prove the following.**

**i.B1 С B2 => f-(B1) С f -( B1) ii. f-(Ui Bi) = Ui f-(Bi) iii. f-(ᴒi Bi) С ᴒi f-(Bi)**

Solution: Here i.B1 С B2 => f-(B1) С f-(B2) Let x ϵ f-(B1) => f(x) ϵ B1 С B2 => x ϵ f- (B1 С B2) =>xϵ f-(B1) С f-(B2) hence B1 С B2 => f-(B1) С f-(B2)

ii. f-(Ui Bi) = Ui f-(Bi) Let xϵ f-(Ui Bi) ⬄ f(x) ϵ UiBi ⬄ f(x) ϵ Bi for some x ⬄ x ϵ f-(Bi) ⬄ xϵ Ui f-(Bi) for some x .Hence f-(Ui Bi) = Ui f-(Bi)

iii. . f-(ᴒi Bi) С ᴒi f-(Bi) Let x ϵ f-(ᴒi Bi) => F(x) ϵ (ᴒi Bi) for all x => f(x) ϵ Bi for all x => x ϵ f-(Bi) for all x

ᵿ x ϵ f-(ᴒi Bi) => x ϵ ᴒi f-(Bi) .Hence f-(ᴒi Bi) С ᴒi f-(Bi) .

**8.Show that the function f(x) = sin(x) is uniformly continuous on [0,∞).**

Solution: Here given function is f(x) = sin(x) ᵿ ϵ > 0 Ʒ a N depending on only ϵ s.t ᵿ n > N on [0,∞)

* |sinn(x) – sin(x) |

= | 2 sin(nx – x)/2 .cos(nx +x)/2|

= 2| sin(nx – x)/2 .cos(nx +x)/2|

≤ 2| sin(nx – x)/2| { cos(nx +x)/2 ≤ 1}

< 2 |n x – x|/2 < n(x – y) < ϵ which is belong to [0,∞) => sin(x) is uniform continous on [0,∞)